Hale School

Mathematics Specialist

Test 5 --- Term 3 2017

Applications of Differentiation and Modelling Motion

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Name: ______________________

Instructions:

- **Calculators are allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Question 1 (4 marks)

Given the equation, $\sqrt{xy} = \ln(\sin y + 2)$ determine $\dfrac{dy}{dx}$.

Question 2 (5 marks)

Find the general solution to the following differential equations:

(a) $\dfrac{dy}{dx} = \cos^2 y$ (2 marks)

(b) $(x^2 - 1)^2 \dfrac{dy}{dx} = \dfrac{2x}{3y}$ (3 marks)

Question 3 (3 marks)

The differential equation for a curve passing through the point (1, -1) is given by $\frac{dy}{dx} = xy - x^2$. Use the incremental formula $\delta y = \frac{dy}{dx} \times \delta x$, with $\delta x = 0.2$, to calculate an estimate for the y-coordinate of the curve when $x = 1.4$.

Question 4 (7 marks)

An object undergoing SHM is defined by the differential equation $\frac{dv}{dt} = -n^2 x$.

(a) Given $\frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$, use integration techniques to show that $v^2 = n^2(A^2 - x^2)$ where A is the amplitude of the motion. (4 marks)

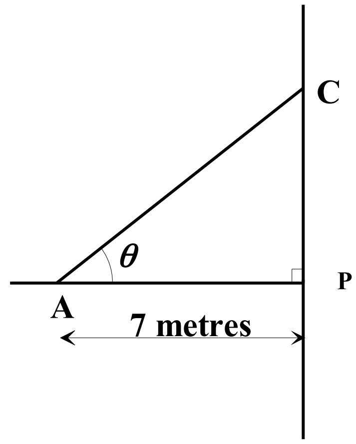
(b) If $x = 4\ m$ when $v = -18\ m/s$ and $x = -3\ m$ when $v = 24\ m/s$, find the period and amplitude of the motion. (3 marks)

Question 5 (8 marks)

A rotating radar gun, at A, for measuring the speed of cars is positioned on a straight section of Great Eastern Highway. The radar gun is set up 7 metres from P, the nearest point on the side of the road, as shown in the diagram. $\angle APC = \frac{\pi}{2}$ and $\angle PAC = \theta^{r}$.

The gun is tracking the car C automatically. It is rotating at 0.358 radians per second at the instant the car is 24 m from P.

(a) Show that the car is exceeding the speed limit of 110 km/hr. (5 marks)

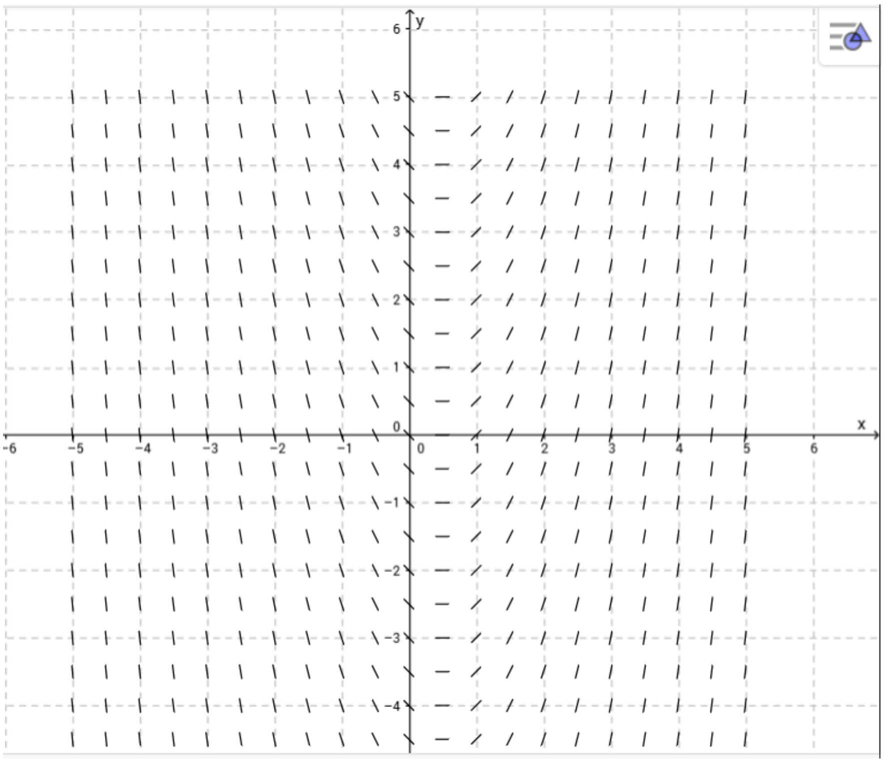


(b) The motorist wants to challenge the fine imposed. After viewing photographic evidence, he suspects that the distance AP was greater than 7 metres. What distance would AP need to be to ensure that the motorist has not exceeded the speed limit.

(3 marks)

Question 6 (7 marks)

The diagram below shows the slope field of a differential equation $\frac{dy}{dx} = f(x, y)$



(a) Determine the general differential equation that would yield this slope field.

(3 marks)

(b) On the slope field given above, draw in a curve representing the particular solution with initial condition (0,-2). (2 marks)

(c) Determine the equation for the particular solution if $\frac{dy}{dx} = -1$ at the point (0,-2) .

(2 marks)

Question 7 (8 marks)

(a) Show that if $P = \dfrac{a}{b + ke^{-at}}$, where a, b and k are positive constants, then $\dfrac{dP}{dt} = aP - bP^2$. (4 marks)

Question 7 continued…

(b) In Dwellingup, it was initially discovered that 10 trees were infected with dieback disease. The resultant growth in the dieback disease is modelled by the equation $\frac{dP}{dt} = 0.1P - 0.000025P^2$, where P is the number of infected trees t months after the initial discovery of the disease.
Use your result from (a) to express P as a function of t. (2 marks)

(c) Calculate the number of months taken for the number of trees infected by dieback to reach 80% of its limiting value. (2 marks)

END OF TEST